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# Electromagnetic Fields In The Ocean Near A Shoreline

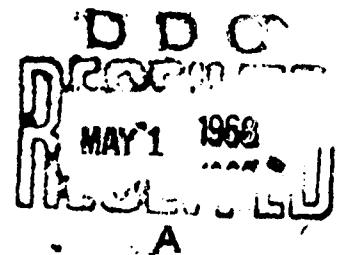
(PART II)

E. SULLIVAN and J. E. SPENCE

Project: Extremely-Low and Sub-Audio Frequency Electromagnetic  
Signals Generated by Natural and Man-Made Effects

Office of Naval Research  
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ELECTROMAGNETIC FIELDS IN THE OCEAN NEAR A SHORELINE

(PART II)

by

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Technical Report 396(10)/3  
Project: Extremely-Low and Sub-Audio Frequency  
Electromagnetic Signals Generated by Natural  
and Man-Made Effects.

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## ABSTRACT

This report represents an extension of the work described in an earlier report [Electromagnetic Fields in the Ocean Near a Shoreline, J.E. Spence, E. Sullivan and J. Beville - Contract Nonr 396(10)-Report 396(10)/2 - 15 November 1965]. It is an analytical study of the electromagnetic field in the ocean resulting from excitation over land by natural electromagnetic noise such as micropulsations or ELF atmospherics. The main purpose of the study is to determine the relative importance of electromagnetic energy entering the ocean via the air-water interface versus energy entering via the soil and ocean bottom. Emphasis in this report is placed on a comparison between the vertical and horizontal components of the electric field. Vertical profiles of the horizontal and vertical electric field vectors are plotted for several values of frequency, ocean depth, and the distance from the shoreline. Finally, a comparison is made with some experimentally determined values of the horizontal electric field.

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## I. INTRODUCTION

Recent experimental measurements of electromagnetic noise in the ocean have sometimes produced data which show an unexpected intensity distribution with depth near a coastline<sup>[12]</sup>. In this thesis the electromagnetic field which may be produced in the ocean by natural sources over land is investigated analytically.

A simplified, two-dimensional model is used which assumes a constant depth ocean with an abrupt shoreline. The field in the ocean is assumed to be excited by electromagnetic energy passing across the coastline, into the sea. The magnetic vector is taken to be parallel to the shoreline.

A superposition method is used which assumes the field in the sea to be composed of two fields - one produced by energy passing through the air-ocean interface and the other produced by energy passing through the soil and ocean bottom. The energy passing through the vertical land-ocean interface is neglected in this treatment.

The waves passing through the air-ocean interface are assumed to be produced by energy which passes across the coastline on the surfaces of the earth and the ocean. The ocean is first assumed to have infinite conductivity which allows the waves to enter the ocean with a direction of propagation normal to the sea surface. The assumption of infinite conductivity is then dropped. The energy passing through the earth-ocean interface is treated in a somewhat similar manner, the difference being that the field intensity at the ocean bottom is determined by an application of Green's integral theorem.

The use of Green's theorem results, as is usually the case, in an integral equation which must be solved by an approximate method. In the present case the solution leads to an integral which is solved by a far-field approximation. The resulting expressions for the electric field components are examined through the use of inequalities and the vertical component of the electric field vector is found to be negligible in comparison to the horizontal component under the restrictions of this model.

The horizontal component of the electric field is plotted for several values of frequency, depth of ocean, and distance from the shoreline. A comparison is then made with some experimentally determined values of the horizontal electric field.

## II. FORMULATION OF THE PROBLEM

The first part of this chapter is concerned with a description of the ocean-shoreline model. The form of the electromagnetic field incident on the shoreline is then presented. Finally, a mathematical expression is obtained which describes the magnetic field in the ocean due to the incident field.

### 2.1 Ocean-Shoreline Model

The geometry of the simplified two-dimensional ocean-shoreline model is shown in Figure 1. In this model the incident field is assumed to have a direction of propagation in the + x direction and all fields are considered to be independent of z, i.e.,  $\partial/\partial z = 0$ .

### 2.2 Incident Field

The magnetic field intensity  $\vec{H}$  will be considered to have a z component only. With this restriction, and the assumption that  $\partial/\partial z = 0$ , Maxwell's curl  $\vec{H}$  equation yields:

$$E_x = \frac{1}{\sigma - i\omega\epsilon} \frac{\partial h}{\partial y} \quad (1a)$$

$$E_y = \frac{-1}{\sigma - i\omega\epsilon} \frac{\partial h}{\partial x} \quad (1b)$$

where

$$\vec{H} = h \vec{u}_z$$

The conductivity and permittivity are given by  $\sigma$  and  $\epsilon$  respectively. The permeability of all media is taken to be  $\mu_0$ , the permeability of free space.

The electromagnetic noise from natural sources over land is assumed to produce a nearly vertically polarized plane wave. A solution to Maxwell's equations for a wave with this polarization propagating over a conducting surface where the direction of propagation is parallel to the surface is well known and is given by<sup>(6)</sup>

$$h = e^{ik_{oy}y + ik_x x} \quad y \geq 0 \quad (2a)$$

$$h = e^{-ik_{2y}y + ik_x x} \quad y \leq 0 \quad (2b)$$

where

$$k_x^2 = \frac{\omega^2 \mu_0 \epsilon_0 - \frac{\omega^2 \epsilon_0}{\sigma_2^2} \left[ i\omega \mu_0 (\sigma_2 - i\omega \epsilon_2) \right]}{1 + \frac{\omega^2 \epsilon_2}{\sigma_2^2}} \quad (3a)$$

$$k_{oy}^2 = \omega^2 \mu_0 \epsilon_0 - k_x^2 \quad (3b)$$

$$k_{2y}^2 = \frac{\omega^2 \mu_0 \epsilon_0 - k_x^2}{i\omega \epsilon_0} (\sigma_2 - i\omega \epsilon_2) \quad (3c)$$

For the frequencies, permittivities, and conductivities considered in this treatment, the quantity  $\omega\epsilon/\sigma \ll 1$ . Under this assumption, Equations (3a), (3b), and (3c) become



$$k_x^2 \approx \frac{\omega^2}{c^2} = k_0^2 \quad (4a)$$

$$k_{oy}^2 \approx \frac{\omega^2}{c^2} \left[ \frac{-i\omega\epsilon_0}{\sigma_2} \right] \quad (4b)$$

$$k_{2y}^2 \approx -i\omega\mu_0\sigma_2 \quad (4c)$$

$$\text{where } c = \frac{1}{\mu_0\epsilon_0}$$

Since  $k_0$  is independent of the conductivity of the guiding surface, the incident magnetic field becomes

$$h_{inc} \approx e^{ik_{oy}y + ik_0x} \quad y \geq 0 \quad (5a)$$

$$h_{inc} \approx e^{-ik_{2y}y + ik_0x} \quad y \leq 0 \quad (5b)$$

where, from Equations (4b) and (4c)

$$k_{oy} = -\frac{\omega}{c} \sqrt{\frac{-i\omega\epsilon_0}{\sigma_2}} \quad (6a)$$

$$k_{2y} = \sqrt{-i\omega\mu_0\sigma_2} \quad (6b)$$

and  $\sigma_2$  is the conductivity of the earth.

### 2.3 Contribution Through the Air-Sea Boundary

In this section an approximate expression for the magnetic field in the ocean due only to the energy entering through the air-sea interface will be determined.

Setting  $y$  equal to zero in Equations (5a) and (5b), it is noted that the incident magnetic field on the surface of the earth is independent of the conductivity of the earth. This is due to the fact that  $k_0$  is independent of the conductivity of the guiding plane. Because of this, propagation in the magnetic field can be considered to be unperturbed at the shoreline. The magnetic field at the top surface of the ocean is then given by

$$h_{ts} \approx e^{ik_0 x} \quad (7)$$

To find the field at some point in the ocean, waves will be considered to enter the water normal to the surface. This assumption is based on the fact that propagation from a poor conductor to a good conductor results in an angle of refraction which is very nearly zero<sup>(14)</sup>. The field at some point in the ocean will thus be of the form

$$h_t \approx B \left[ e^{-ik_1(y+d)} + p_t e^{ik_1(y+d)} \right] \quad (8)$$

where

$$k_1 = \sqrt{\mu_0 \left( \epsilon_1 - \frac{\sigma_1^2}{i\omega} \right)} \approx \sqrt{i\omega\mu_0\sigma_1}$$

and  $\sigma_1$  is the conductivity of the sea water. It is to be noted that  $d$ , the depth of the ocean, has been introduced into the exponents in Equation (8). This is done in the interest of simplifying the final expression. The subscript  $t$  indicates that this is the magnetic field in the ocean due to the energy entering through the top surface only. The quantity  $\rho_b$  is given by<sup>(8)</sup>

$$\rho_b = \frac{\sqrt{\frac{\mu_o}{\epsilon_1 - \frac{\sigma_1}{i\omega}}} - \sqrt{\frac{\mu_o}{\epsilon_2 - \frac{\sigma_2}{i\omega}}}}{\sqrt{\frac{\mu_o}{\epsilon_1 - \frac{\sigma_1}{i\omega}}} + \sqrt{\frac{\mu_o}{\epsilon_2 - \frac{\sigma_2}{i\omega}}}} \approx \frac{\sqrt{\frac{\sigma_2}{\sigma_1}} - 1}{\sqrt{\frac{\sigma_2}{\sigma_1}} + 1}$$

The conductivities of the earth and sea water are given to be  $10^{-3}$  mhos/meter and either 3 or 4 mhos/meter respectively<sup>(12)</sup>. With these values  $\rho_b$  is very nearly equal to -1. Continuity of  $h$  across the boundary at  $y = 0$  requires that

$$B = \frac{h_{ts}}{e^{-ik_1 d} + \rho_b e^{ik_1 d}}$$

The final expression for  $h_t$  then becomes

$$h_t \approx \left[ \frac{e^{-ik_1(y+d)} + \rho_b e^{ik_1(y+d)}}{e^{-ik_1 d} + \rho_b e^{ik_1 d}} \right] h_{ts} \quad (9)$$

#### 2.4 Contribution Through the Land-Sea Boundary

In this section the magnetic field in the sea due only to the energy entering through the earth-ocean boundary is determined.

Consider an arbitrary closed surface  $s'$  bounding a source free region. It can be shown that a field inside this region at some point  $r$  is given by (9)

$$\psi(r) = -\frac{1}{i\omega\epsilon} \oint_{s'} \left[ G(r|r') \frac{\partial}{\partial n'} \psi(r') - \psi(r') \frac{\partial}{\partial n'} G(r|r') \right] ds' \quad (10)$$

where  $G(r|r')$  is known as the Green's function which is the solution to the inhomogeneous Helmholtz equation

$$(\nabla^2 + k^2) G(r|r') = i\omega\epsilon \delta(|\vec{r} - \vec{r}'|); \quad (11)$$

$\delta(|\vec{r} - \vec{r}'|)$  is the Dirac delta function,  $r'$  is the coordinate of a point source on  $s'$ ,  $r$  is the field coordinate, and the operator  $\partial/\partial n'$  denotes the derivative normal to the surface  $s'$ .

Referring to Figure 2, it can be seen that a closed surface  $s'$  can be selected such that some of its elements are parallel to the  $z$  axis and the remaining surface elements are parallel to the  $x, y$  plane. The surface areas made up of elements parallel to the  $x, y$  plane are outlined by the contour  $L'$ . Setting  $\psi = h$  it is noted that since  $\partial/\partial z = 0$ , the normal derivatives of  $\psi(r')$  and  $G(r|r')$  with respect to  $r'$  vanish on all elements of  $s'$  parallel to the  $x, y$  plane. As a result of this, Equation (10) reduces to a line integral along  $L'$ , i.e.,

$$h(r) = -\frac{1}{i\omega\epsilon_2} \oint_{\mathcal{L}'} \left[ G(r|r') \frac{\partial}{\partial n'} h(r') - h(r') \frac{\partial}{\partial n'} G(r|r') \right] d\mathcal{L}' \quad (12)$$

where

$$r = \sqrt{x^2 + y^2}, \quad r' = \sqrt{x'^2 + y'^2}$$

and  $x'$  and  $y'$  are simply the rectangular coordinates of the point source on the surface  $s'$ .

The Green's function will be chosen such that its normal derivative will vanish along  $y = -d$  for all  $x$ . In addition, it is again temporarily assumed that the ocean is a perfect conductor such that  $\partial h / \partial n'$  vanishes for  $y = -d$ ,  $x \neq 0$ . Under these restrictions the integral along  $y = -d$ ,  $0 \leq x \leq +\infty$  is zero. Also, by Equations (3a) and (3c), it can be seen that the integral along the infinite arc is also zero due to finite losses. Thus Equation (10) reduces to an integral along the  $y$  axis from  $y = -d$  to  $y = +\infty$ .

The solution to Equation (10) subject to the restriction that  $\partial / \partial y' G(r|r') = 0$  at  $y = -d$  is well known and is given by<sup>(13)</sup>

$$G(r|r') = \frac{\omega\epsilon_2}{2} H_0^{(1)} \left[ k_2 \sqrt{(x-x')^2 + (y'+d)^2} \right] \quad (13)$$

where  $H_0^{(1)} \left[ k_2 \sqrt{(x-x')^2 + (y'+d)^2} \right]$  is the zeroth order Hankel function of the first kind and

$$k_2 = \omega \sqrt{\mu_0 \left( \epsilon_2 - \frac{\sigma_2}{i\omega} \right)} \approx \sqrt{1\omega\mu_0\sigma_2}$$

This Green's function represents the magnetic field at  $r$  due to a line source at  $r'$  where the line source is described by the Dirac delta function  $\delta(y-y')$  times  $i\omega\epsilon_2$ . Its physical counterpart would be a magnetic line current source.

In order to evaluate Equation (12) it is necessary to know the magnetic field and its derivative with respect to  $x'$  along  $y'$ , the path of the integration. This, of course, is not known exactly. As is often the case in problems of this type, we shall approximate  $h$  over this path by assuming that it is equal to the incident magnetic field which is given by Equation (5b)<sup>(3)</sup>. Substitution of Equation (5b) and Equation (13) into Equation (11) yields

$$h_{bs} \approx \int_{-d}^{\infty} \frac{-e^{-ik_2 y}}{2} \left( k_x H_0^{(1)} \left[ k_2 \sqrt{x^2 + (y' + d)^2} \right] + \frac{ik_2 x}{\sqrt{x^2 + (y' + d)^2}} H_1^{(1)} \left[ k_2 \sqrt{x^2 + (y' + d)^2} \right] \right) dy' \quad (14)$$

This integral cannot be readily evaluated for all values of  $x$ . However, for sufficiently large  $x$ , say  $x > 5 \delta_2$ , where  $\delta_2 = \frac{2}{\omega \mu_0 \sigma_2}$  is the skin depth in the earth, it is shown in the appendix that Equation (14) can be approximated by

$$h_{bs} \approx \sqrt{\frac{i}{2\pi k_2 x}} e^{ik_2(x+d)} \quad (15)$$

The assumption is now made that waves enter the ocean in a direction normal to the bottom surface. This, as before, is justified by the fact that

$\sigma_1 \gg \sigma_2$ . The field at some point in the ocean then becomes

$$h_b \approx C \left[ e^{ik_1 y} + \rho_t e^{-ik_1 y} \right]$$

The subscript b indicates that this is the magnetic field in the ocean due to the energy entering through the bottom of the ocean only.  $\rho_t$  is the reflection coefficient at the top of the ocean and is given by

$$\rho_t = \frac{\sqrt{\frac{\mu_o}{\epsilon_1 - \frac{\sigma_1}{i\omega}}} - \sqrt{\frac{\mu_o}{\epsilon_o}}}{\sqrt{\frac{\mu_o}{\epsilon_1 - \frac{\sigma_1}{i\omega}}} + \sqrt{\frac{\mu_o}{\epsilon_o}}}$$

This reflection coefficient is also very nearly equal to -1. Requiring  $h$  to be continuous across the boundary yields

$$C = \frac{h_{bs}}{e^{-ik_1 d} + \rho_t e^{+ik_1 d}}$$

The final expression for  $h_b$  now becomes

$$h_b \approx \left[ \frac{e^{+ik_1 y} + \rho_t e^{-ik_1 y}}{e^{-ik_1 d} + \rho_t e^{+ik_1 d}} \right] h_{bs} \quad (16)$$

This derivation has neglected the vertical portion of the earth-ocean interface. This is justified by the fact that the skin depth in the ocean turns out to be always considerably less than the shortest distance from

shore for which the final expression is valid.

## 2.5 Total Magnetic Field in the Ocean

The total magnetic field in the ocean is given by the sum of Equation (9) and Equation (16) which gives

$$h_{\text{total}} = \left[ \frac{\frac{-ik_1(y+d)}{e} + \rho_b e}{\frac{-ik_1 d}{e} + \rho_b e} \right] h_{ts} + \left[ \frac{\frac{ik_1 y}{e} + \rho_t e}{\frac{-ik_1 d}{e} + \rho_t e} \right] h_{bs} \quad (17)$$

Setting  $\rho_b$  and  $\rho_t$  equal to -1 allows Equation (17) to be reduced to

$$h_{\text{total}} = \frac{\text{Sink}_1(y+d)}{\text{Sink}_1 d} h_{ts} - \frac{\text{Sink}_1 y}{\text{Sink}_1 d} h_{bs} \quad (18)$$

where  $h_{ts}$  and  $h_{bs}$  are given by Equation (7) and Equation (15) respectively.



### III. THE ELECTRIC FIELD

The vertical and horizontal components of the electric field are found in the first part of this chapter. The two field components are then compared with respect to their relative magnitudes.

#### 3.1 The Electric Field Components

Now that an expression for  $h$  has been obtained, the electric field is easily obtained by substituting Equation (18) into Equations (1a) and (1b). The horizontal component of the electric field thus becomes

$$E_x = \frac{k_1}{\sigma_1} \left[ \frac{\cos k_1(y+d)}{\sin k_1 d} h_{ts} - \frac{\cos k_1 y}{\sin k_1 d} h_{bs} \right] \quad (19)$$

and the vertical component becomes

$$E_y = \frac{-1}{\sigma_1} \left[ \frac{\sin k_1(y+d)}{\sin k_1 d} \frac{\partial h_{ts}}{\partial x} - \frac{\sin k_1 y}{\sin k_1 d} \frac{\partial h_{bs}}{\partial x} \right] \quad (20)$$

where, again, use has been made of the fact that  $\omega \epsilon_1 / \sigma_1 \ll 1$ .

#### 3.2 Comparison of the Field Components

Although the vertical and horizontal components of the electric field have been determined, the direction of the net electric vector is not immediately apparent. In this section the magnitudes of the two components of the electric vector will be compared.

Consider first the problem of the infinitely deep ocean. Due to the absence of the earth-ocean interface, there would be no energy radiated

in the  $xy$  direction. The solution would then reduce to Equation (5b) with  $k_{2y}$  replaced by  $k_1$ . The magnetic field would then become

$$h_{\text{total}} \approx e^{-ik_1 y + ik_0 x} \quad (21)$$

where, as before, the contribution through the vertical earth-ocean interface has been ignored. From Equations (1a) and (1b), the electric field components now become

$$E_x = -i \frac{k_1}{\sigma_1} h_{\text{total}} \quad (22a)$$

$$E_y = -i \frac{k_0}{\sigma_1} h_{\text{total}} \quad (22b)$$

Dividing Equation (22b) by (22a) and taking the absolute magnitude yields

$$\left| \frac{E_y}{E_x} \right| = \left| \frac{k_0}{k_1} \right| = \sqrt{\frac{\omega \epsilon_0}{\sigma_1}} \quad (23)$$

Setting  $\epsilon_0 = 8.85 \times 10^{-12}$  farads/meter and  $\sigma_1 = 4$  mhos/meter, Equation (23) becomes

$$\left| \frac{E_y}{E_x} \right| \approx 1.5 \sqrt{\omega} \times 10^{-6}$$

It is apparent, therefore, that for frequencies of  $10^{10}$  c/s and less, the vertical component of the electric field may be neglected in comparison to its horizontal component.

This result is no surprise since it already has been pointed out that the waves are entering the ocean with a direction of propagation normal to the surface. It is not so obvious, however, that this result

should hold for an ocean with a finite depth for it might be argued that, at some point, the upward traveling waves and the downward traveling waves might have x components of their respective electric fields which combine destructively whereas their y components combine constructively.

It shall now be shown that the vertical component can still be neglected under the restrictions of the model used in this study.

Since no values of  $d$  less than 5 ocean skin depths are used, the approximation is made that reflections at the earth-ocean and air-ocean interfaces may be neglected. The magnetic field may then be written as

$$h \approx h_{ts} e^{-ik_1 y} + h_{bs} e^{ik_1(y+d)} \quad (24)$$

Substitution of Equations (7) and (15) into Equation (24) yields

$$h \approx e^{ik_0 x - ik_1 y} - \sqrt{\frac{1}{2\pi k_2 x}} e^{ik_2(x+d) + ik_1(y+d)}$$

Use of Equations (1a) and (1b) then give the electric field as

$$E_x \approx -i \frac{k_1}{\sigma_1} \left[ e^{-ik_1 y} + \sqrt{\frac{1}{2\pi k_2 x}} e^{ik_2(x+d) + ik_1(y+d)} \right] \quad (25a)$$

$$E_z \approx -\frac{1}{\sigma_1} \left[ ik_0 e^{-ik_1 y} - ik_2 \left( 1 - \frac{1}{2k_2 x} \right) \sqrt{\frac{1}{2\pi k_2 x}} e^{ik_2(x+d) + ik_1(y+d)} \right] \quad (25b)$$

where, since  $k_0 \ll k_2$ ,  $k_2$  has been neglected. Taking the absolute

magnitude of the ratio of  $E_y$  to  $E_x$  results in

$$\left| \frac{E_y}{E_x} \right| = \left| \frac{\frac{k_0}{k_1} - \frac{k_2}{k_1} \sqrt{\frac{1}{2\pi k_2 x}} e^{ik_2(x+d) + ik_1(2y+d) - ik_0 x}}{1 + \sqrt{\frac{1}{2\pi k_2 x}} e^{ik_2(x+d) + ik_1(2y+d) - ik_0 x}} \right| \quad (26)$$

where use has been made of the fact that

$$1 - \frac{1}{2\pi k_2 x} \approx 1$$

Defining

$$M = |E_y/E_x| \left/ \left| \frac{k_2}{k_1} \right| \right.$$

Equation (26) is rewritten as

$$M = \left| \frac{S - Ae^{i\theta}}{1 + Ae^{i\theta}} \right| \quad (27)$$

where

$$A = \left| \sqrt{\frac{1}{2\pi k_2 x}} \right| \quad (28a)$$

$$\theta = \frac{\pi}{8} + (1+i) \left| \frac{k_2}{\sqrt{2}} \right| (x+d) + (1+i) \left| \frac{k_1}{\sqrt{2}} \right| (2y+d)$$

$$- \left( \left| \frac{k_2}{\sqrt{2}} \right| (x+d) + \left| \frac{k_1}{\sqrt{2}} \right| (2y+d) + \frac{\pi}{8} \right) \quad (28b)$$

$$+ i \left( \left| \frac{k_2}{\sqrt{2}} \right| (x+d) + \left| \frac{k_1}{\sqrt{2}} \right| (2y+d) \right) - \alpha + i\beta$$

$$S = k_0/k_2 \quad (28c)$$

Equation (27) now becomes

$$M = \left| \frac{S - A [\cos(\alpha + i\beta) + i \sin(\alpha + i\beta)]}{1 + A [\cos(\alpha + i\beta) + i \sin(\alpha + i\beta)]} \right| \quad (29)$$

Using the identities

$$\cos(\alpha + i\beta) = \cos\alpha \cosh\beta - i \sin\alpha \sinh\beta$$

$$\sin(\alpha + i\beta) = \sin\alpha \cosh\beta + i \cos\alpha \sinh\beta$$

Equation (28) can be rewritten as

$$M = \left| \frac{\{S_R - A \cos\alpha [\cosh\beta - \sinh\beta]\} + i \{S_I + A \sin\alpha [\sinh\beta - \cosh\beta]\}}{\{1 + A \cos\alpha [\cosh\beta - \sinh\beta]\} - i \{A \sin\alpha [\sinh\beta - \cosh\beta]\}} \right| \quad (30)$$

where  $S_R + i S_I = S$

Equation (30) can be further simplified by making the substitution

$$\pm [\cosh\beta - \sinh\beta] = \pm e^{-\beta}$$

Then Equation (30) finally becomes

$$M = \left| \frac{\{S_R - A \cos\alpha e^{-\beta}\} + i \{S_I - A \sin\alpha e^{-\beta}\}}{\{1 + A \cos\alpha e^{-\beta}\} + i \{A \sin\alpha e^{-\beta}\}} \right| \quad (31)$$

In order to reduce the complexity of Equation (31) the restriction that  $\beta \geq 0$  will be imposed and the consequences of this restriction will later be examined.  $\beta$  is defined by Equation (28b).

Setting  $\beta = 0$  and noting that for the parameters used in this model  $A \leq 0.15$ , the denominator of Equation (31) can now be represented in the inequality

$$\left| \{ 1 + A \cos \alpha \} + i \{ A \sin \alpha \} \right| \geq (1 - 0.15) = 0.85$$

Under the same restrictions, the numerator has an upper bound given by

$$\left| \{ S_R - A \cos \alpha \} + i \{ S_I - A \sin \alpha \} \right| \approx \left| A (\cos \alpha + i \sin \alpha) \right| \leq \sqrt{2} (0.15) = 0.212$$

where, since  $S_I = S_R = 5.2 \times 10^{-3}$ , they have been neglected. Under the restriction then that  $\beta = 0$ , the maximum value that  $|E_y/E_x|$  can take on is given by

$$\left| \frac{E_y}{E_x} \right| = M \left| \frac{k_2}{k_1} \right| = M \sqrt{\frac{\sigma_2}{\sigma_1}} \leq \sqrt{\frac{\sigma_2}{\sigma_1}} \left( \frac{0.212}{0.85} \right) \quad (32)$$

Now since  $A \leq 0.15$ , it is obvious from Equation (31) that Equation (32) holds also for  $\beta > 0$ . Setting  $\sigma_1 = 4$  mhos/meter and  $\sigma_2 = 10^{-3}$  mhos/meter the inequality (32) becomes

$$\left| \frac{E_y}{E_x} \right| \leq 4 \times 10^{-3}$$

In order to geometrically define the region in which this inequality holds, Equation (29b) is solved for  $y$  with  $\beta = 0$ . This results in

$$y = -\frac{d}{2} - \frac{1}{2} \left| \frac{k_2}{k_1} \right| (x+d) = -\frac{d}{2} - \frac{1}{2} \sqrt{\frac{\sigma_2}{\sigma_1}} (x+d) \quad (33)$$

Since the smallest value of  $x$  used is very much larger than the largest value of  $d$  used, Equation (33) can be approximated by

$$y \approx -\frac{d}{2} - \frac{1}{2} \sqrt{\frac{\sigma_2}{\sigma_1}} x \quad (34)$$

Equation (34) is plotted in Figure 3. It can be seen that the inequality (32) holds for all values of  $x$  and  $y$  except for the cross hatched region near the shoreline.

Since  $\sigma_1$  and  $\sigma_2$  are assumed in this model to be  $\sigma_1 = 10^{-3}$  mhos/meter and  $\sigma_2 = 3-4$  mhos/meter, the  $x$  intercept becomes  $x = 62.5 d$  for  $\sigma_2 = 4$  mhos/meter and  $x = 54.1d$  for  $\sigma_2 = 3$  mhos/meter. For the purpose of considering  $|E_y/E_x|$  in the cross hatched region, Equation (27) is written in a slightly different form. Noting that

$$|e^{i\theta}| = e^{-\theta}$$

Equation (27) can be rewritten as an inequality, i.e.,

$$M \leq \frac{Ae^{-\theta}}{|1-Ae^{-\theta}|} = M'$$

As a trial, the requirement is made that  $|Ae^{-\theta}| \geq 1.1$  in order to avoid a singularity. Under this restriction  $M' \leq 11$  and hence

$$\left| \frac{E_y}{E_x} \right| \leq \sqrt{\frac{\sigma_2}{\sigma_1}} (11) = 1.74 \times 10^{-1}$$

Now the smallest value of  $A$  encountered in this treatment is 0.097.

Then the relation

$$(0.097)e^{-\theta} = 1.1$$

yields a minimum of -2.42 for  $\beta$ . Hence it is now only necessary to consider  $\beta$  in the range  $0 \geq \beta \geq -2.42$ .

As  $\beta$  ranges from 0 to -2.42,  $e^{-\beta}$  ranges from 1 to 11.3 and  $\cos\alpha$  ranges from 0.96 to -0.276 and passes through zero only once. The lower bound of the denominator of Equation (31) is then given by

$$\left| \{ 1 + A \cos\alpha e^{-\beta} \} \right| \geq 1 + A (11.3) (-0.276)$$

but  $A \leq 0.15$ , then

$$\left| \{ 1 + A \cos\alpha e^{-\beta} \} \right| \leq 1 - 0.466 = 0.534$$

The numerator of Equation (31) has an upper bound given by

$$\left| A \cos\alpha e^{-\beta} - i A \sin\alpha e^{-\beta} \right| \leq \sqrt{2} (0.15) (11.3)$$

where, as before,  $S_R$  and  $S_I$  have been neglected. It is concluded then, that when  $0 \geq \beta \geq -2.42$ ,

$$M \leq \frac{\sqrt{2} (0.15) (11.3)}{0.534} = 4.5$$

or

$$\left| \frac{E_y}{E_x} \right| \leq (4.5) \sqrt{\frac{\sigma_2}{\sigma_1}} = 7.2 \times 10^{-2}$$

The restrictions on these results, that is that outside the cross hatched region  $|E_y/E_x| \leq 4 \times 10^{-3}$  and inside the cross hatched region  $|E_y/E_x| \leq 7.2 \times 10^{-2}$ , are first, that the ocean depth be at least 58,



so as to validate the assumption that reflections at the earth-ocean and air-ocean interfaces are negligible, and secondly that

$$0.097 \leq A \leq 0.15$$

This is equivalent to requiring that

$$5\delta_2 \leq x \leq 12\delta_2$$

For  $x$  less than  $5\delta_2$ , the far field approximation given by Equation (15) no longer holds and for  $x$  greater than  $12\delta_2$ , the contribution through the earth-ocean interface is eliminated to such an extent that the profiles approach a simple exponential form.

The variation of  $A$  with  $\sigma_2$  is extremely slow, such that a change in  $\sigma_2$  of plus or minus an order of magnitude has a negligible effect on these results.

Comparing the upper bounds on  $|E_y/E_x|$  it is noted that inside the cross hatched region in Figure 3, the upper bound is a factor of 18 larger than the upper bound outside of the cross hatched region. This points out the possibility that very near the shore, in the region where this model is no longer valid,  $|E_y|$  may be of an appreciable size with respect to  $|E_x|$ .

#### IV. NUMERICAL RESULTS

Numerical values for the magnitude of the  $x$  and  $y$  components of the electric field were determined by digital evaluation of Equation (19). The field is normalized by multiplying by  $|\sigma_1/k_1|$ . The results are plotted in Figures 4 to 16 as a function of depth and distance from shore for frequencies of 10, 60, 100, and 1000 cycles/sec. Depths range from 50 meters to 800 meters and distances from shore range from 3 km to 50 km. The depth into the ocean is plotted as the abscissa and the various values of  $x$  (distance from shore) determine a family of curves.

In Figure 17 a comparison is made with some experimental points determined at a frequency of 60 cycles/sec. and a depth of 300 meters<sup>(12)</sup>.

The value of  $x$  has a lower limit which is determined by the approximation used to evaluate the integral in Equation (14). This is essentially a far-field approximation which requires  $x$  to be no smaller than about  $5\delta_2$  where  $\delta_2 = \sqrt{\frac{2}{\omega\mu_0\sigma_2}}$  is the skin depth of the earth.  $\delta_1$ , the skin depth in the ocean, and  $\delta_2$  are plotted for convenience as a function of frequency in Figures 19 and 18 respectively.

The contribution to the field due to the energy passing through the air-ocean boundary only is shown as a dotted line. This enables one to see more explicitly the effect that the energy passing through the horizontal land-ocean interface has on the total field.

## V. SUMMARY AND CONCLUSIONS

The electromagnetic field in the ocean resulting from excitation by natural sources over land has been investigated with the intent of gaining some insight as to the mechanism by which the energy actually enters the water. The horizontal and vertical components of the electric field are plotted for several values of the parameters involved and, in one case, a comparison is made with some experimental measurements.

Before drawing any conclusions it should be stated that both, the land-sea model and the configuration of the incident field, which have been chosen are only a very rough approximation of the real physical situation. Nevertheless, the approximations used are felt to be justified by both, the complex nature of the problem and the reasonable agreement with some experimental results.

Upon examination of the curves, it is seen that the contribution to the net field which enters through the land-ocean interface is quite prominent in the region near the shoreline. As the distance from shore increases, this contribution becomes less and less noticeable. This is due to the fact that  $k_x$ , the wave number which governs the propagation in the  $x$  direction for the contribution through an air-ocean interface, has a small imaginary part, whereas the attenuation of the contribution through the land-ocean interface is quite high due to the conductivity of the earth.

The effect of the depth of the ocean on the electric field is also quite noticeable. For the region near the shoreline the curves show the

bottom contribution more distinctly as the depth of the ocean increases. This is understandable since for a deep ocean the energy entering the top surface will be greatly attenuated before reaching the bottom. The contribution through the bottom will therefore be more in evidence.

The comparison with experiment made in Figure 17 is rather hampered by a lack of data points. It is to be noted, however, that an increase in the field intensity is clearly evident in the experimental results. The discrepancy between the experimental points and the theoretical curves could be due to any or all of the following: The crudeness of the mathematical model, differences between actual and used values for conductivity, conductivity gradients in the earth, and reflections from boundaries in the earth below the ocean bottom.

In future work a more complicated model is to be considered. More specifically, the earth beneath the ocean will be assumed to be composed of two layers. It is hoped that the model will give even better agreement with experimental results. Considering the problem in reverse, this double-layer model could produce a method for determining not only the conductivity of the earth under the ocean, but also the thickness of the top layer.

## APPENDIX A

Far Field Approximation for  $h_{bs}$ 

Rewriting Equation (14) in terms of skin depths results in

$$h_{bs} = \int_{-d}^{-\infty} \frac{-c}{2} \frac{1}{\delta_2} \left( k_x H_0^{(1)} \left[ k_2 \sqrt{x^2 + (y'+d)^2} \right] + \frac{ik_2 x}{\sqrt{x^2 + (y'+d)^2}} H_1^{(1)} \left[ k_2 \sqrt{x^2 + (y'+d)^2} \right] \right) dy' \quad (35)$$

Because of the decaying exponential character of the integrand, its contribution to the integral is appreciable only for that portion of the path of integration within a few skin depths of the ocean bottom. Equation (35) may then be rewritten as

$$h_{bs} = \int_{-d}^{-\infty} f(x, y') dy' \approx \int_{-d}^{-2\delta_2} f(x, y') dy'$$

where  $f(x, y')$  is the integrand in Equation (35).

If the values of  $x$  are now restricted to be greater than  $5\delta_2$ , the argument of the Hankel functions can be greatly simplified over that portion of the path of integration that is most important, i.e.

$$\sqrt{x^2 + (y' + d)^2} \approx x \quad \begin{array}{l} x \geq 5\delta_2 \\ y' \leq 2\delta_2 \end{array}$$

Since the values of  $d$  considered are much less than  $\delta_2$ , its presence in the argument does not affect the approximation to any great extent.

With these approximations, the Hankel functions are no longer functions of  $y'$  and therefore may be moved outside of the integral sign leaving a simple exponential to integrate. Upon integrating, Equation (35) becomes

$$h_{bs} \approx \frac{ie^{ik_2 d}}{2k_2} \left[ k_x H_0^{(1)}(k_2 x) + ik_2 H_1^{(1)}(k_2 x) \right] \quad (36)$$

Factoring  $ik_2 H_1^{(1)}(k_2 x)$  outside the brackets results in

$$h_{bs} \approx -\frac{1}{2} e^{ik_2 d} H_1^{(1)}(k_2 x) \left[ 1 - i \frac{k_x}{k_2} \frac{H_0^{(1)}(k_2 x)}{H_1^{(1)}(k_2 x)} \right] \quad (37)$$

The large argument approximations for the Hankel functions are given as (4)

$$H_0^{(1)}(k_2 x) = \sqrt{\frac{2}{\pi k_2 x}} e^{i[k_2 x - \frac{\pi}{4}]} \quad (38a)$$

$$H_1^{(1)}(k_2 x) = \sqrt{\frac{2}{\pi k_2 x}} e^{i[k_2 x - \frac{3\pi}{4}]} \quad (38b)$$

Dividing Equation (33a) by Equation (33b) yields

$$\frac{H_0^{(1)}(k_2 x)}{H_1^{(1)}(k_2 x)} \approx e^{i \frac{\pi}{2}} = i \quad \text{for large } k_2 x$$

This considerably simplifies Equation (32) i.e.,

$$h_{bs} \approx \sqrt{\frac{1}{2\pi k_2 x}} e^{ik_2(x+d)} \left[ 1 + \frac{k_x}{k_2} \right] \quad (39)$$

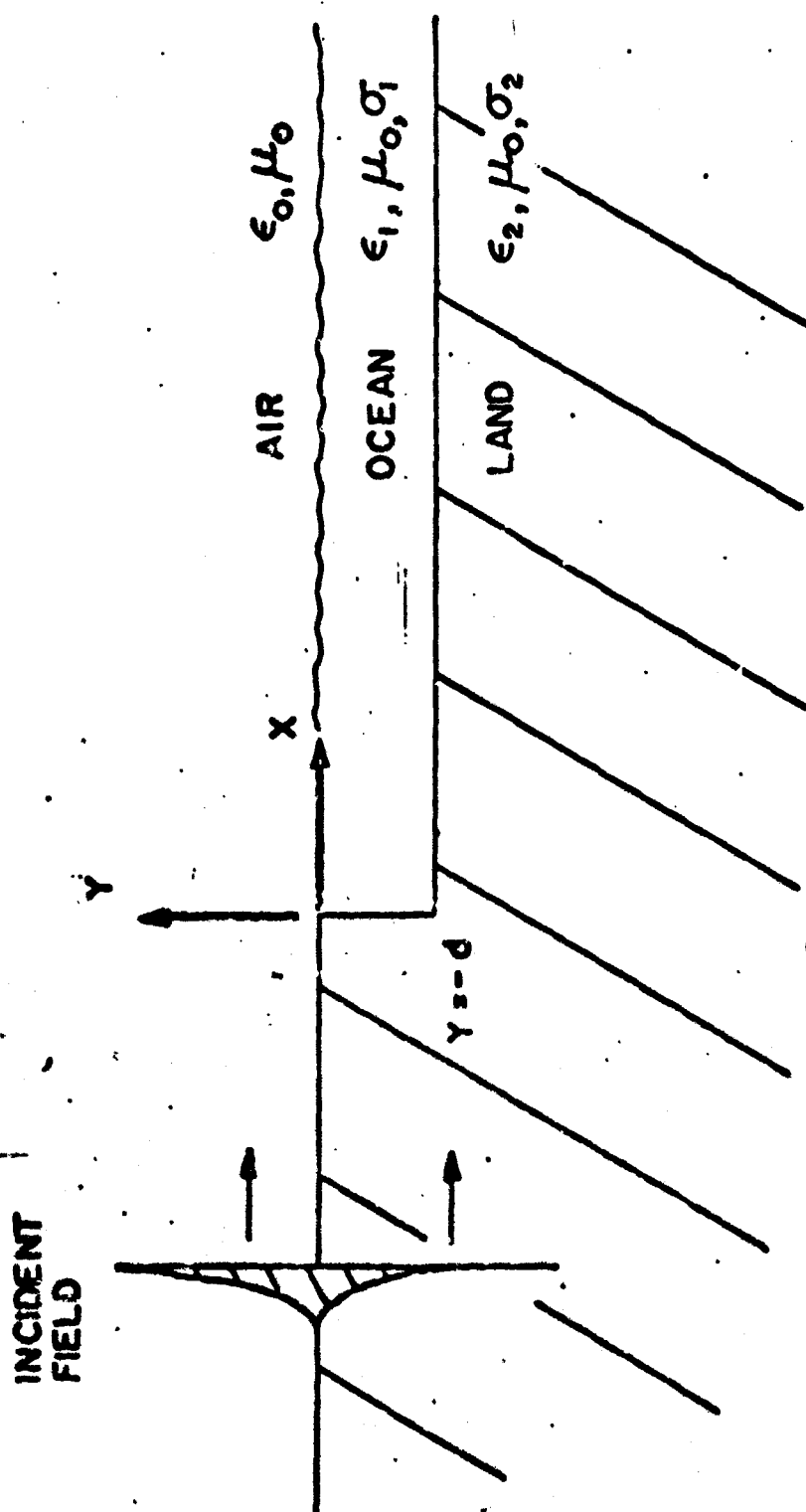
Noting that  $k_x \ll k_2$ , Equation (39) finally becomes

$$h_{bs} \approx \sqrt{\frac{1}{2\pi k_2 x}} e^{ik_2(x+d)}$$

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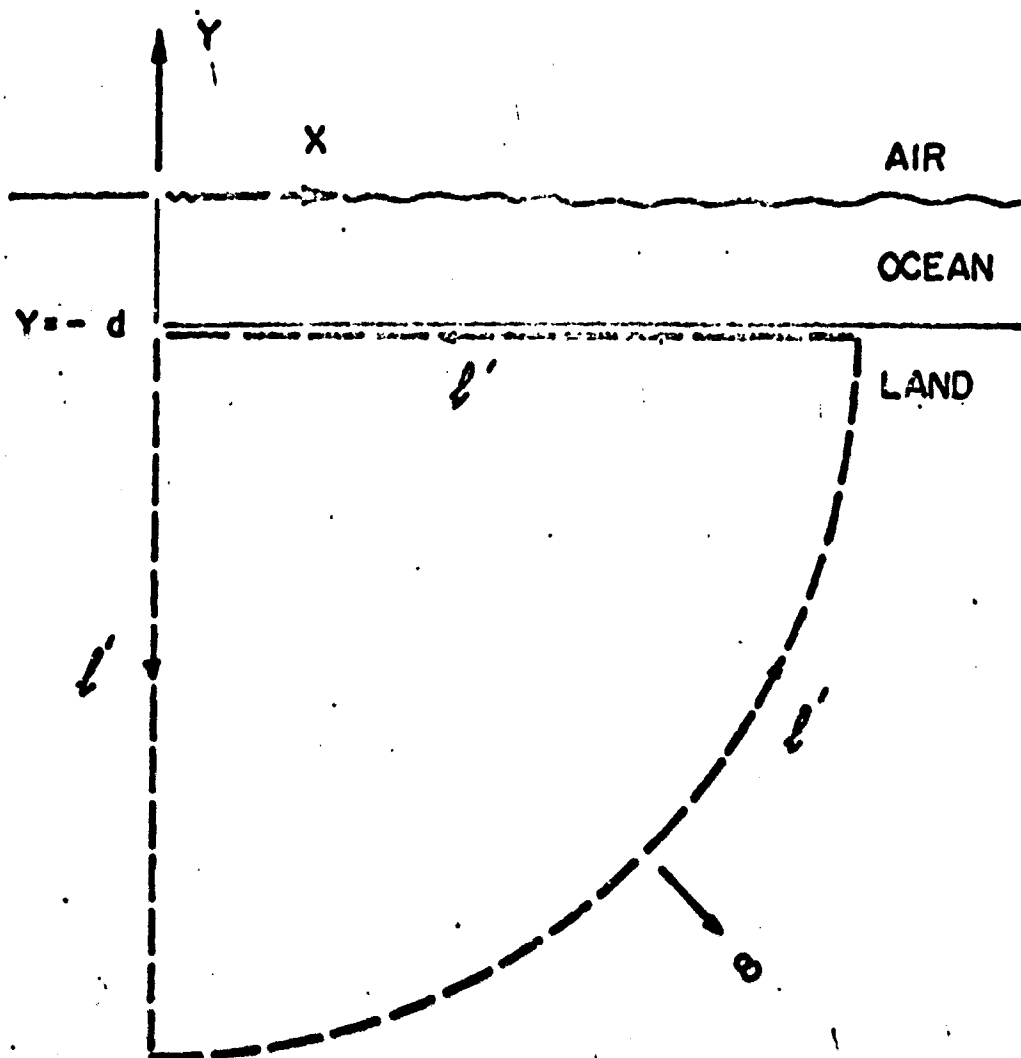
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SIMPLIFIED TWO DIMENSIONAL MODEL

FIGURE - 1



REGION TO WHICH GREEN'S THEOREM  
IS APPLIED

FIGURE-2



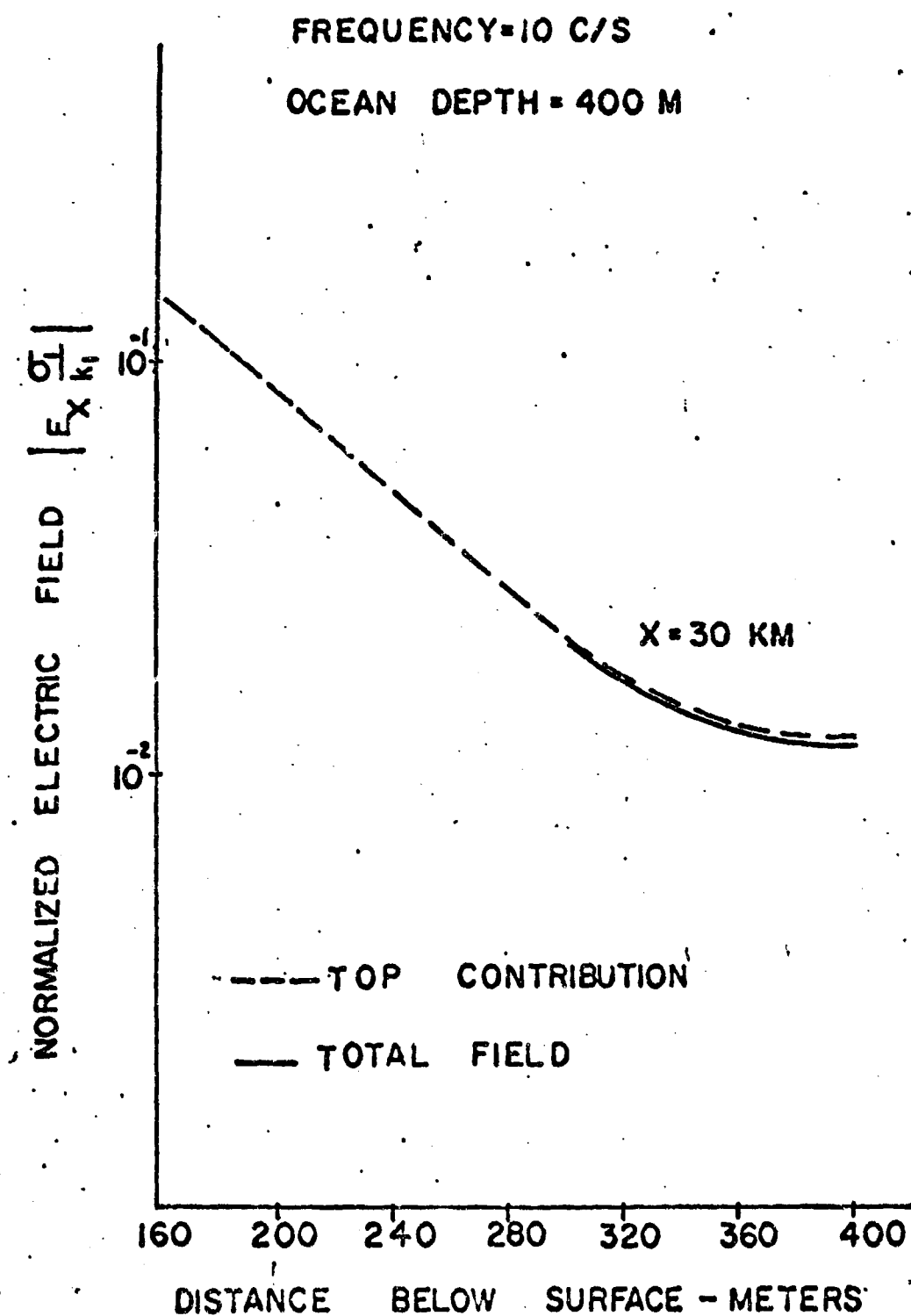


FIGURE - 4

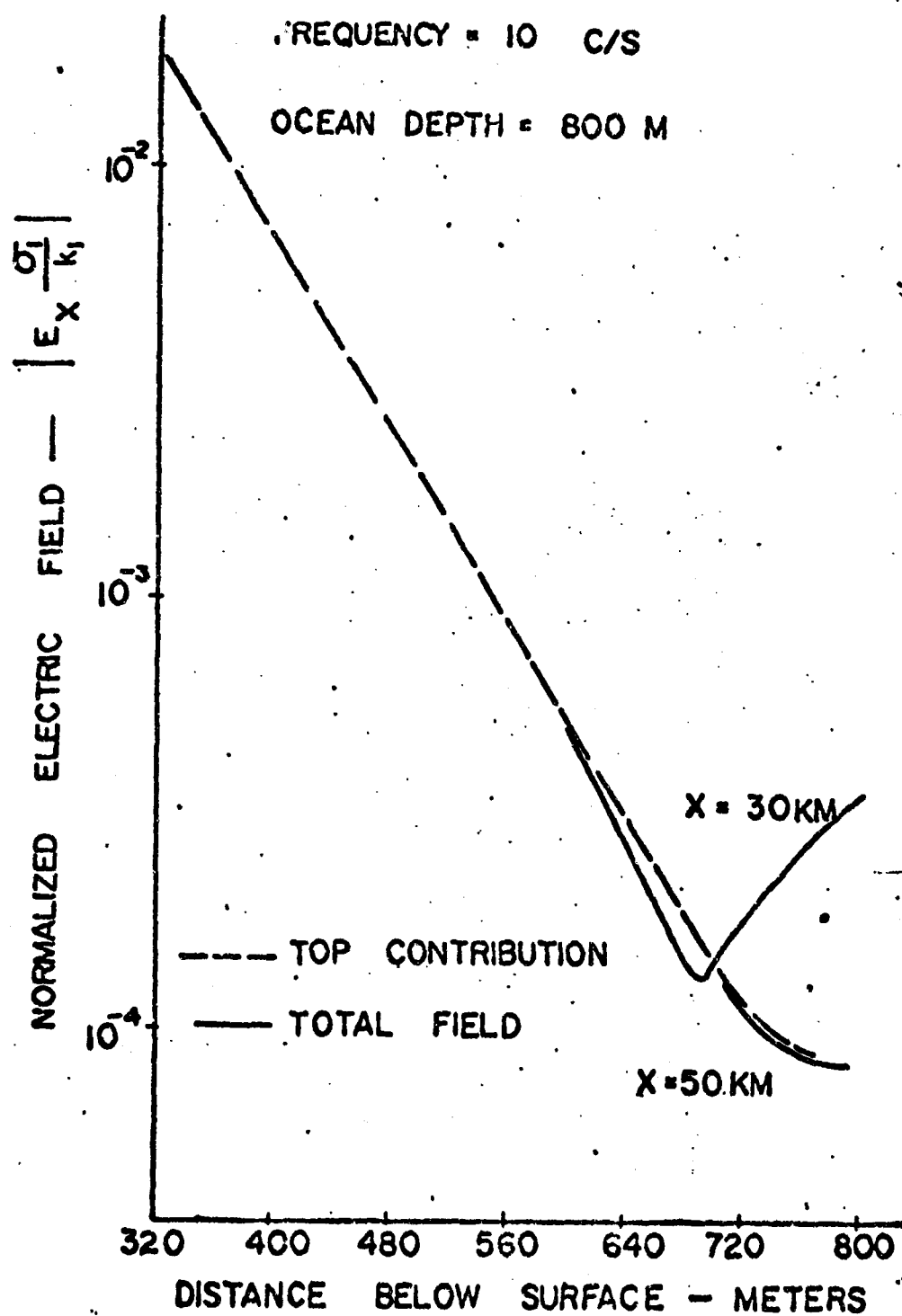


FIGURE - 5

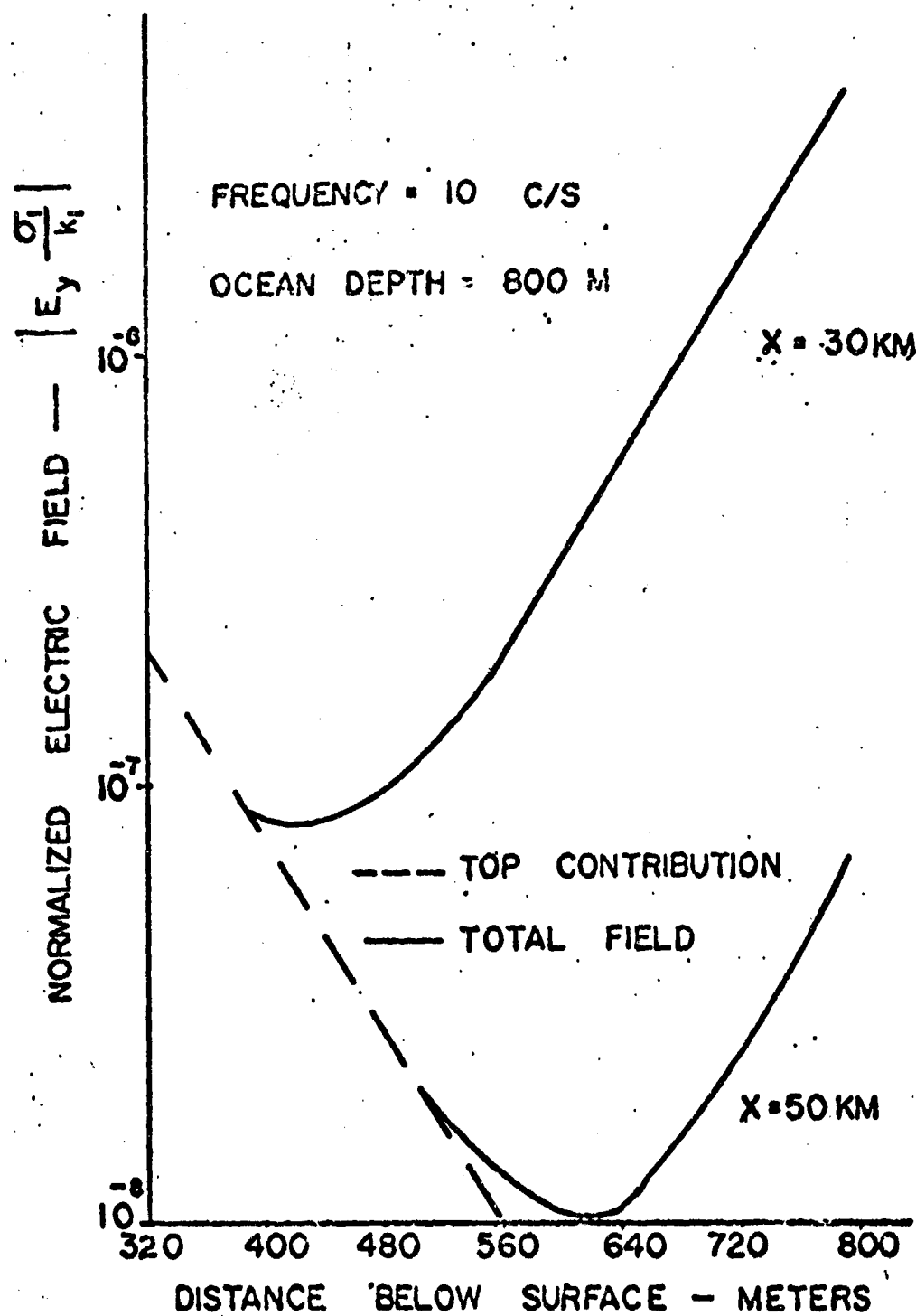


FIGURE - 6

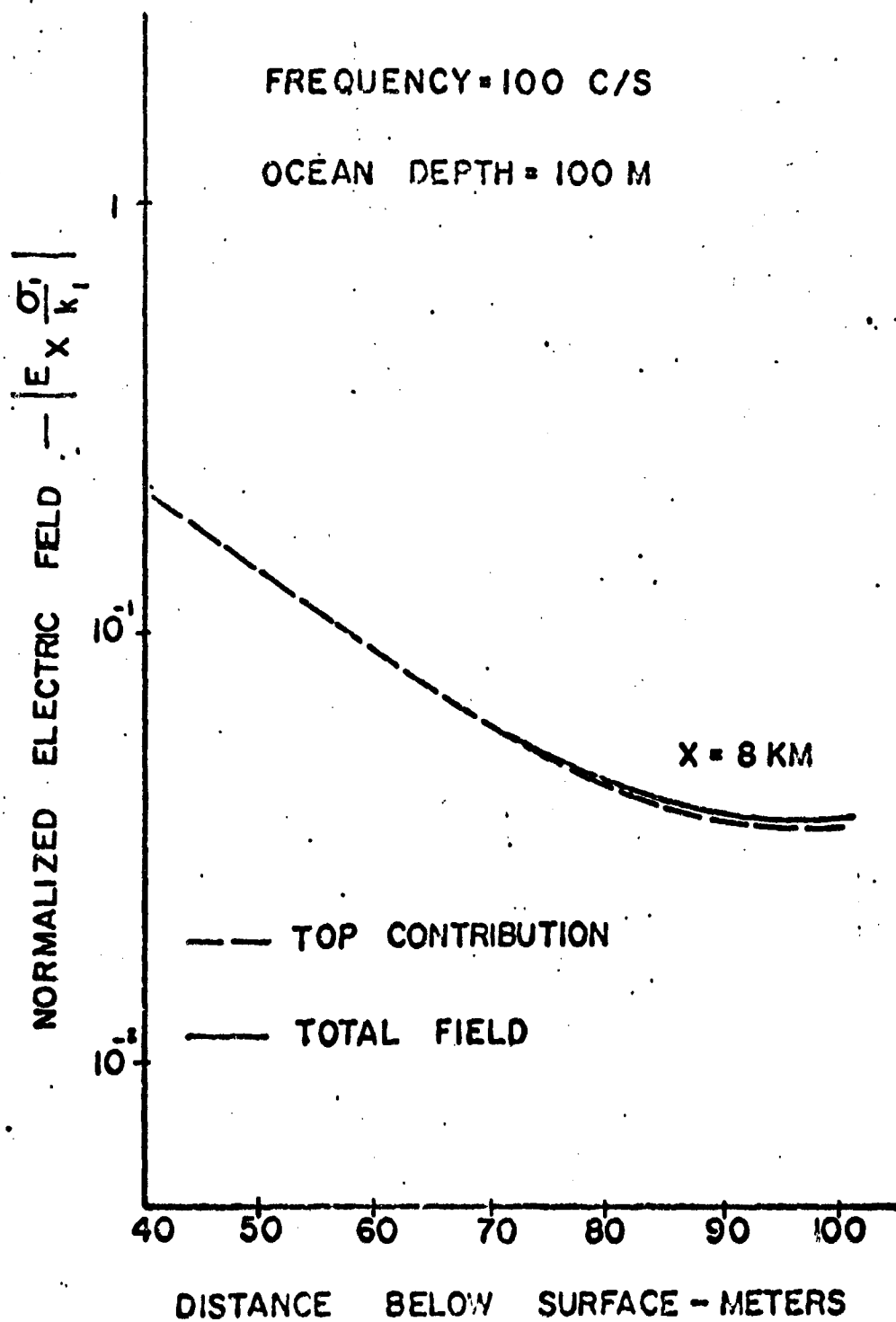


FIGURE - 7

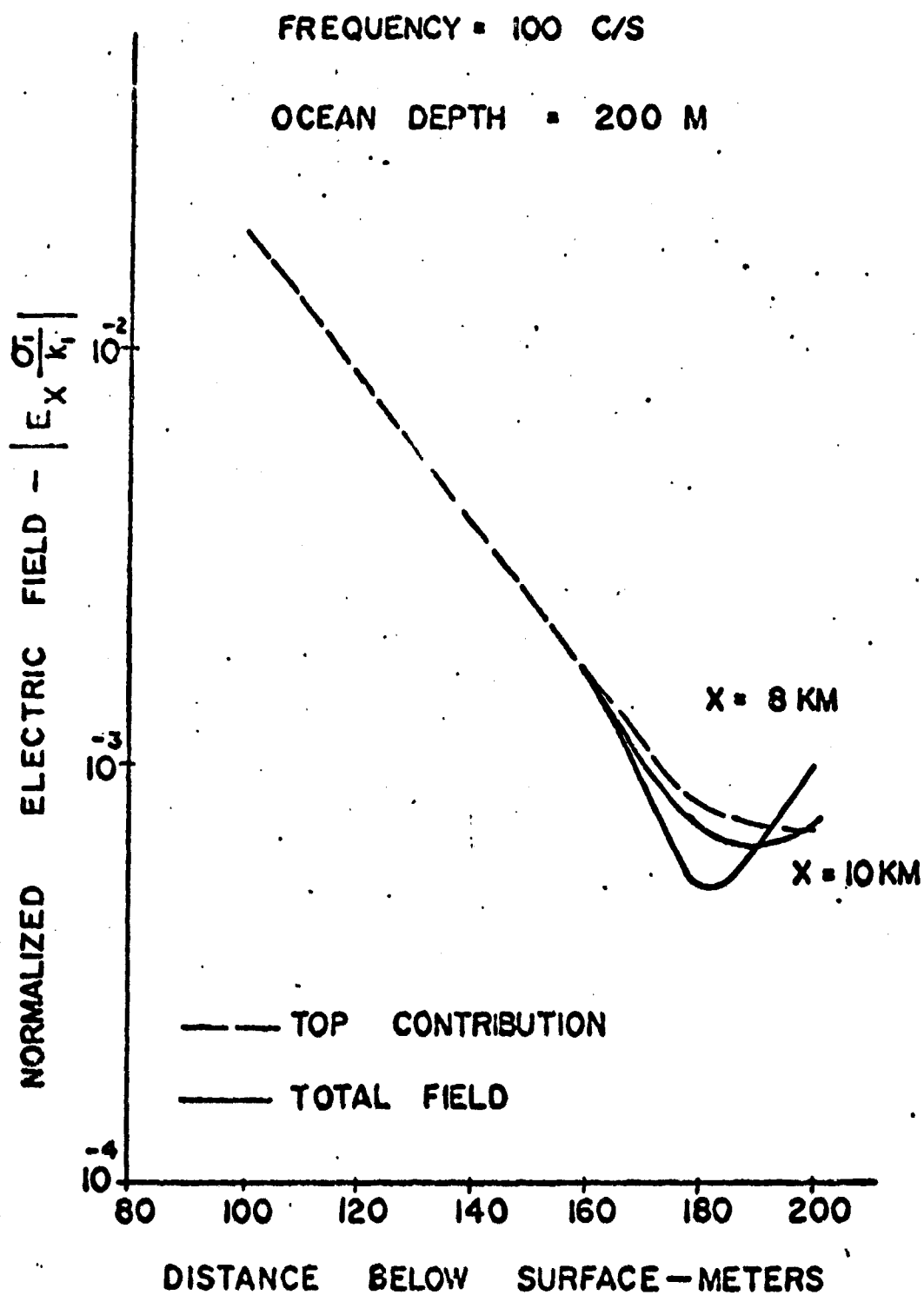


FIGURE - 8.



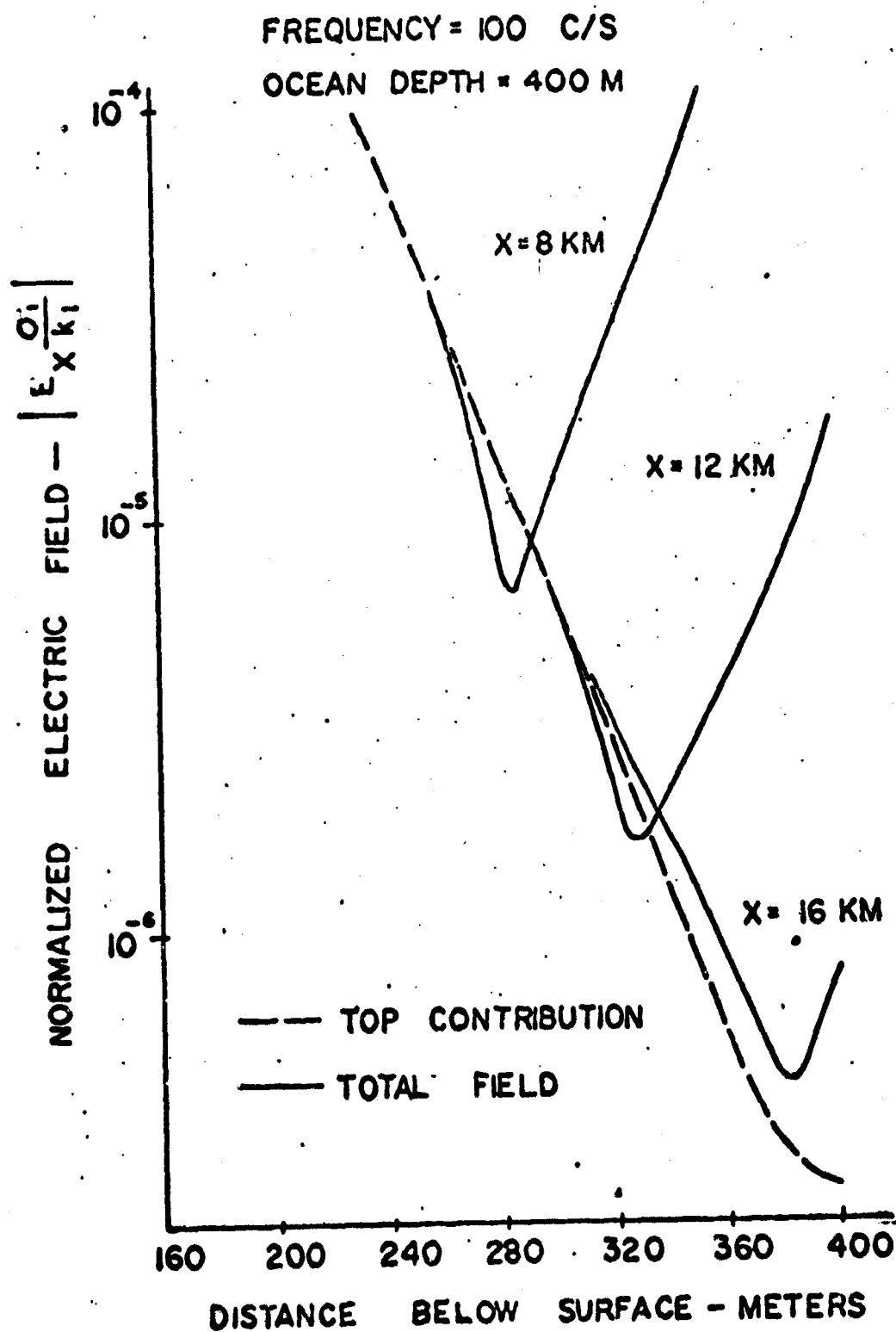


FIGURE - 9

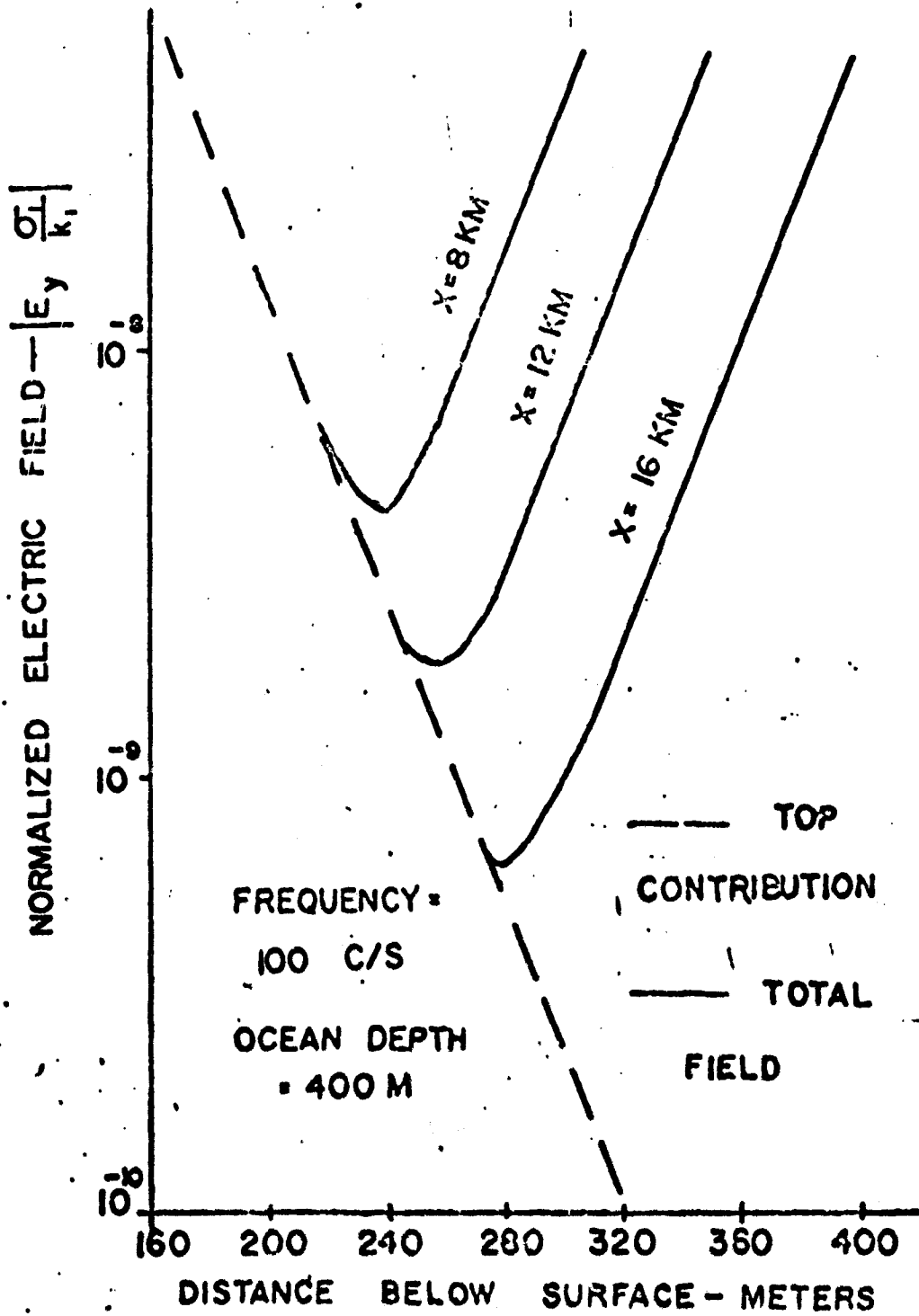


FIGURE -10

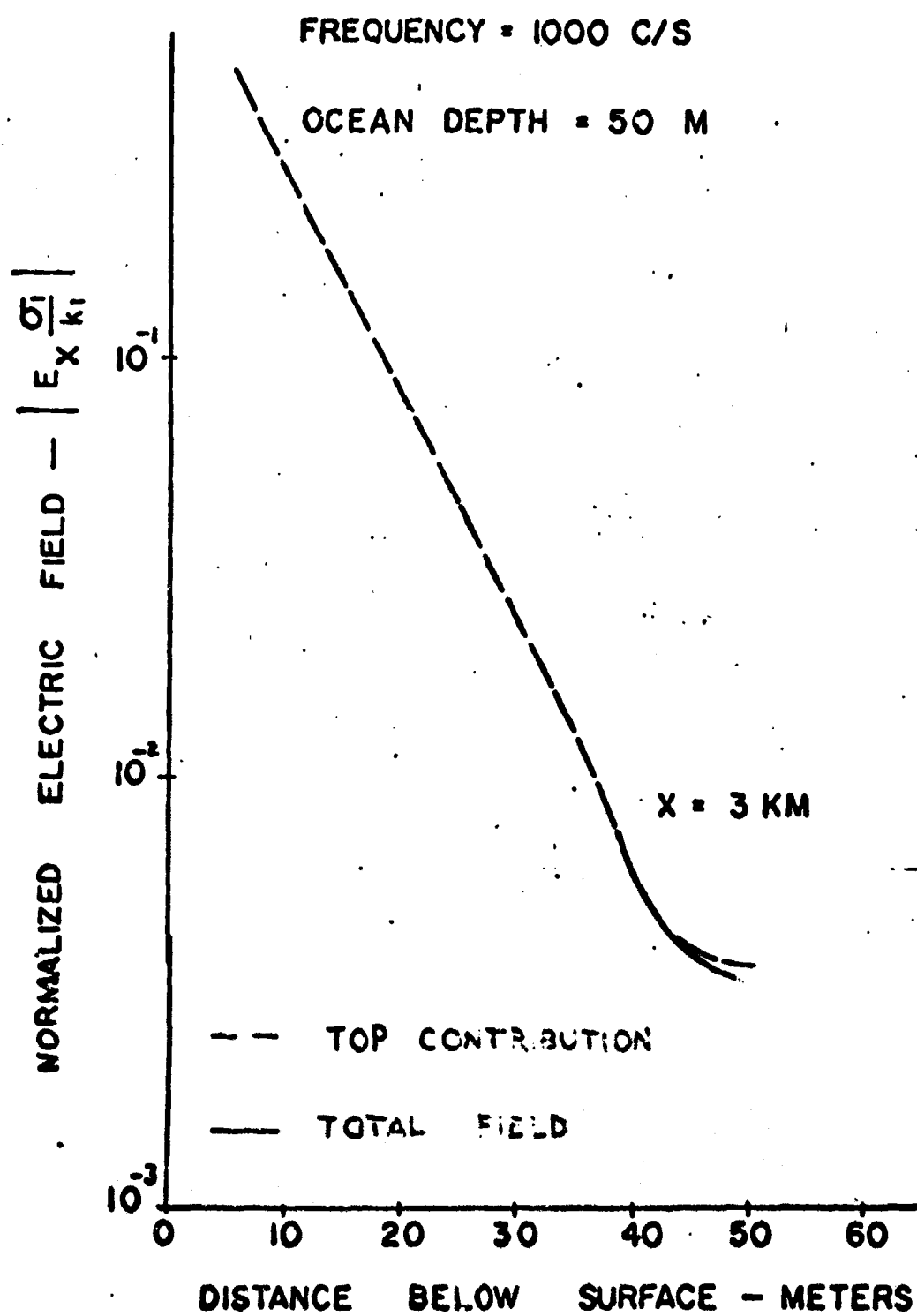


FIGURE - 11

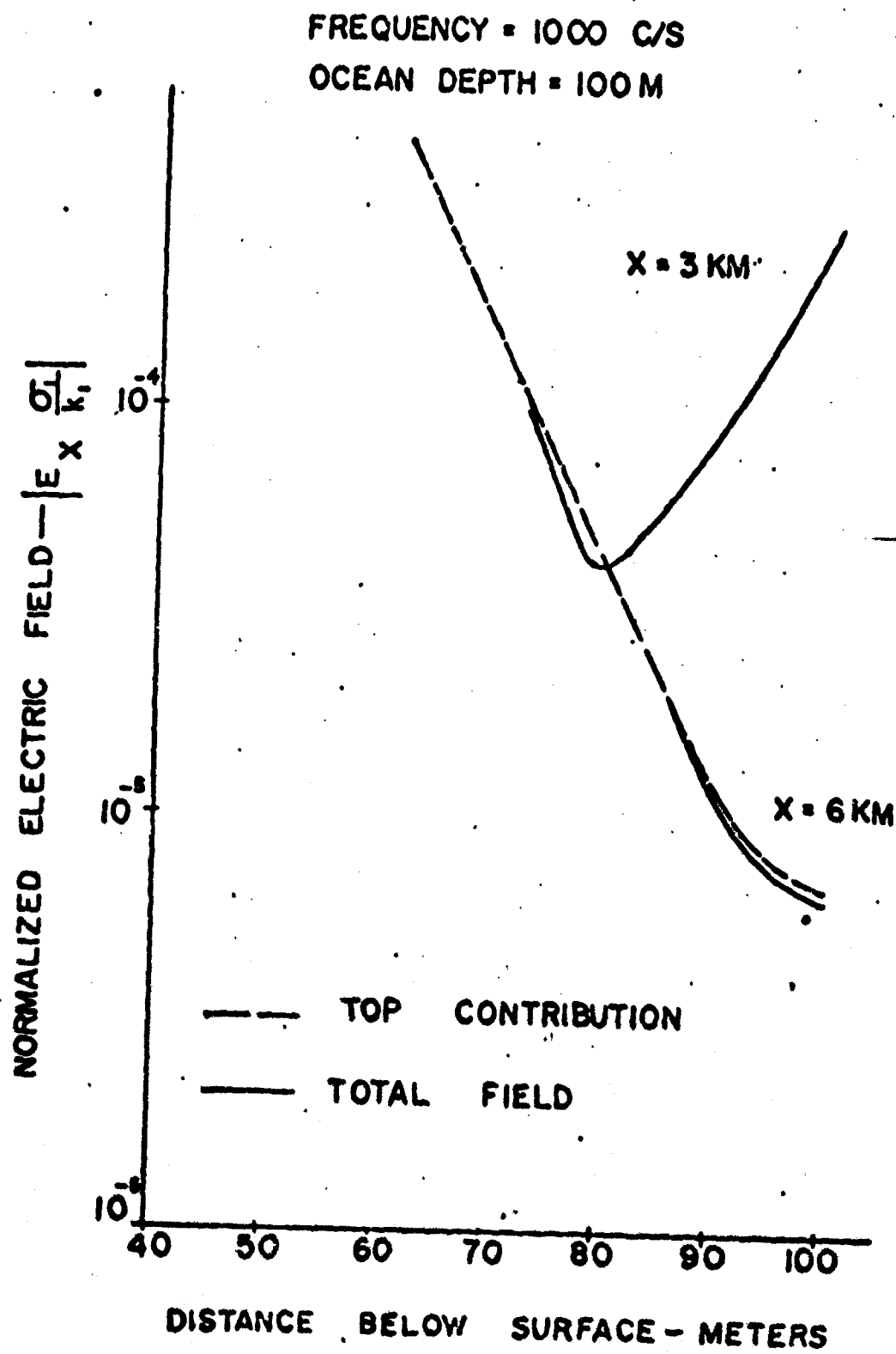


FIGURE - 12

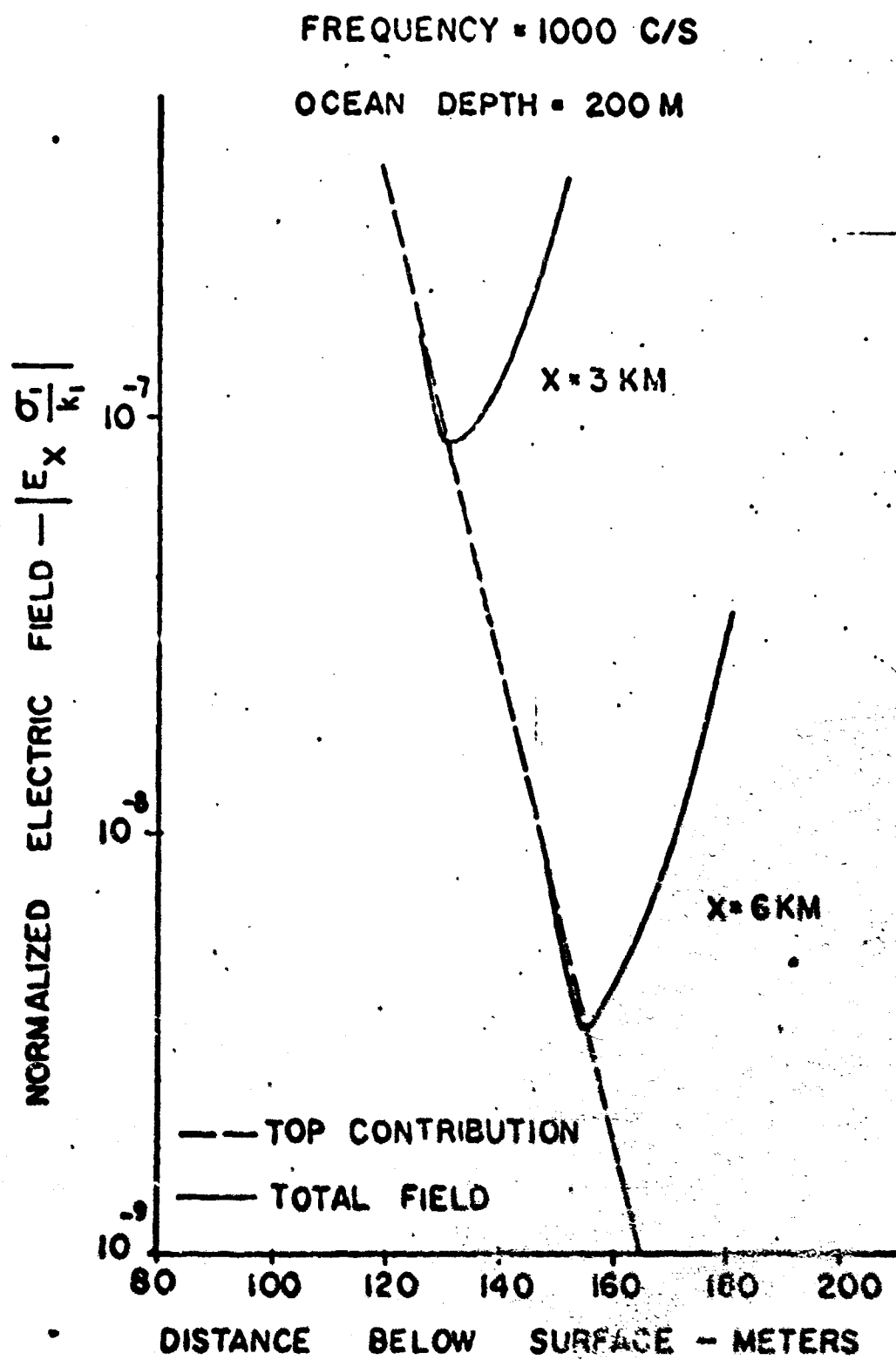


FIGURE - 13

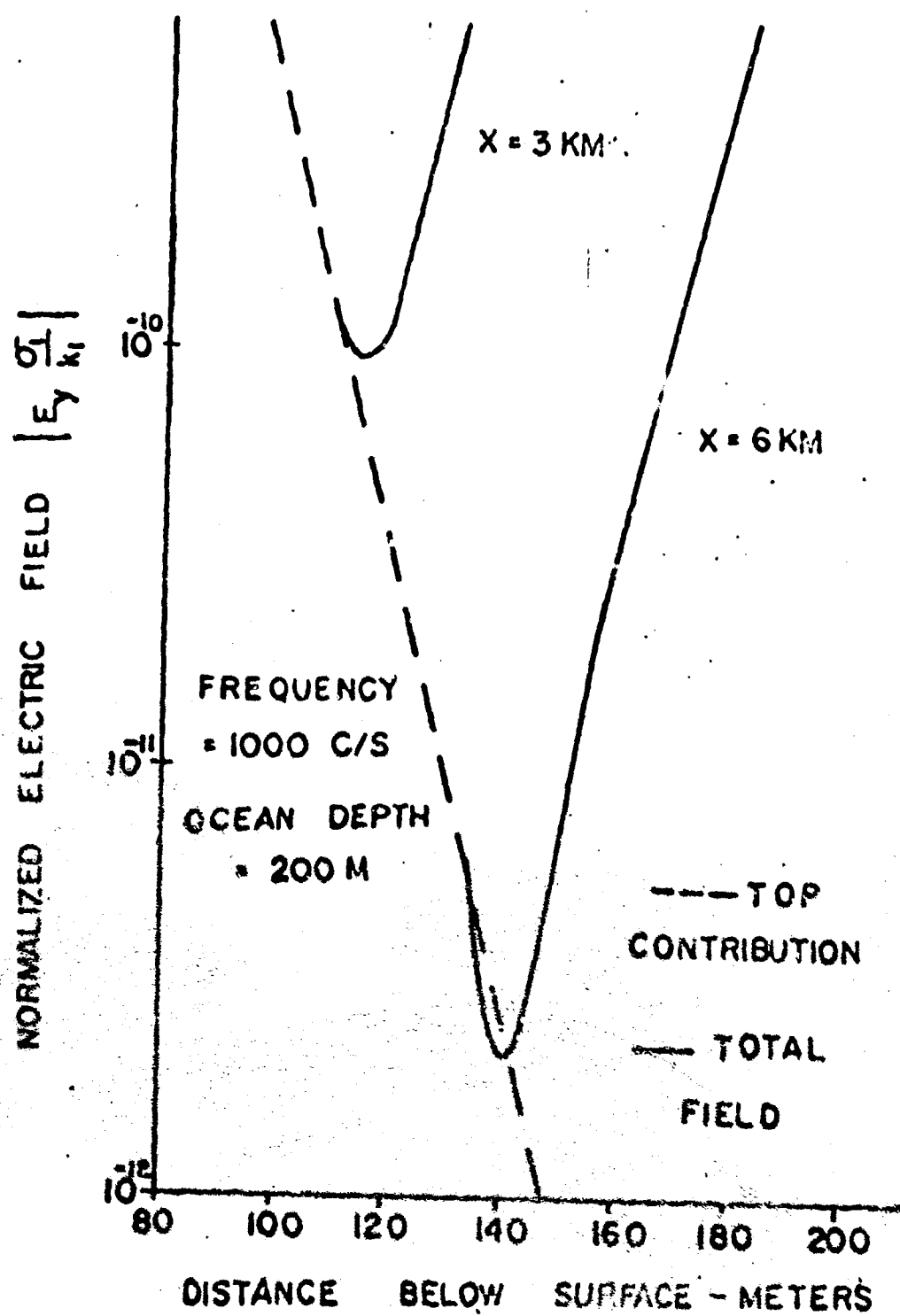


FIGURE -14

FREQUENCY = 1000 C/S

OCEAN DEPTH = 400 M

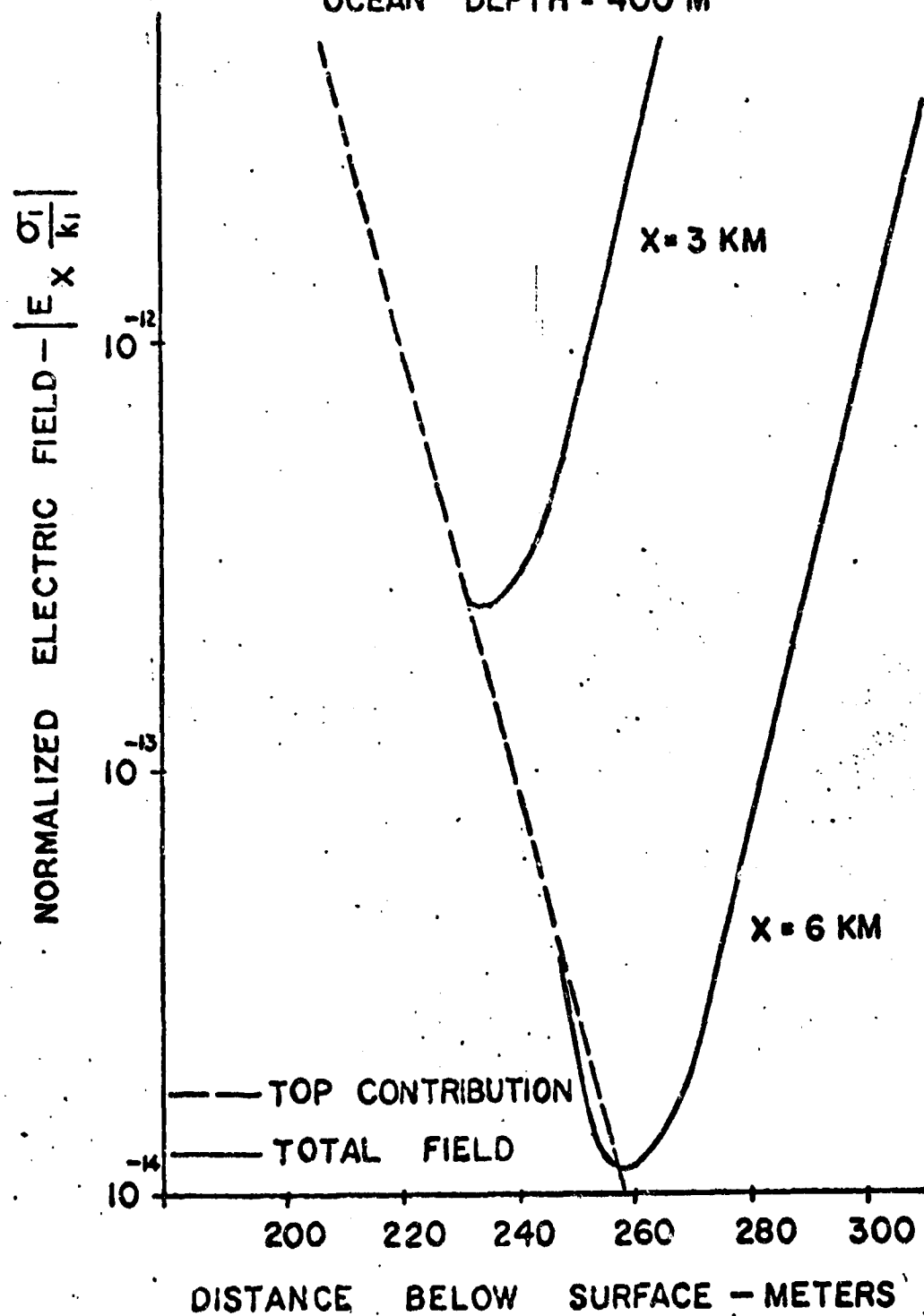


FIGURE - 15

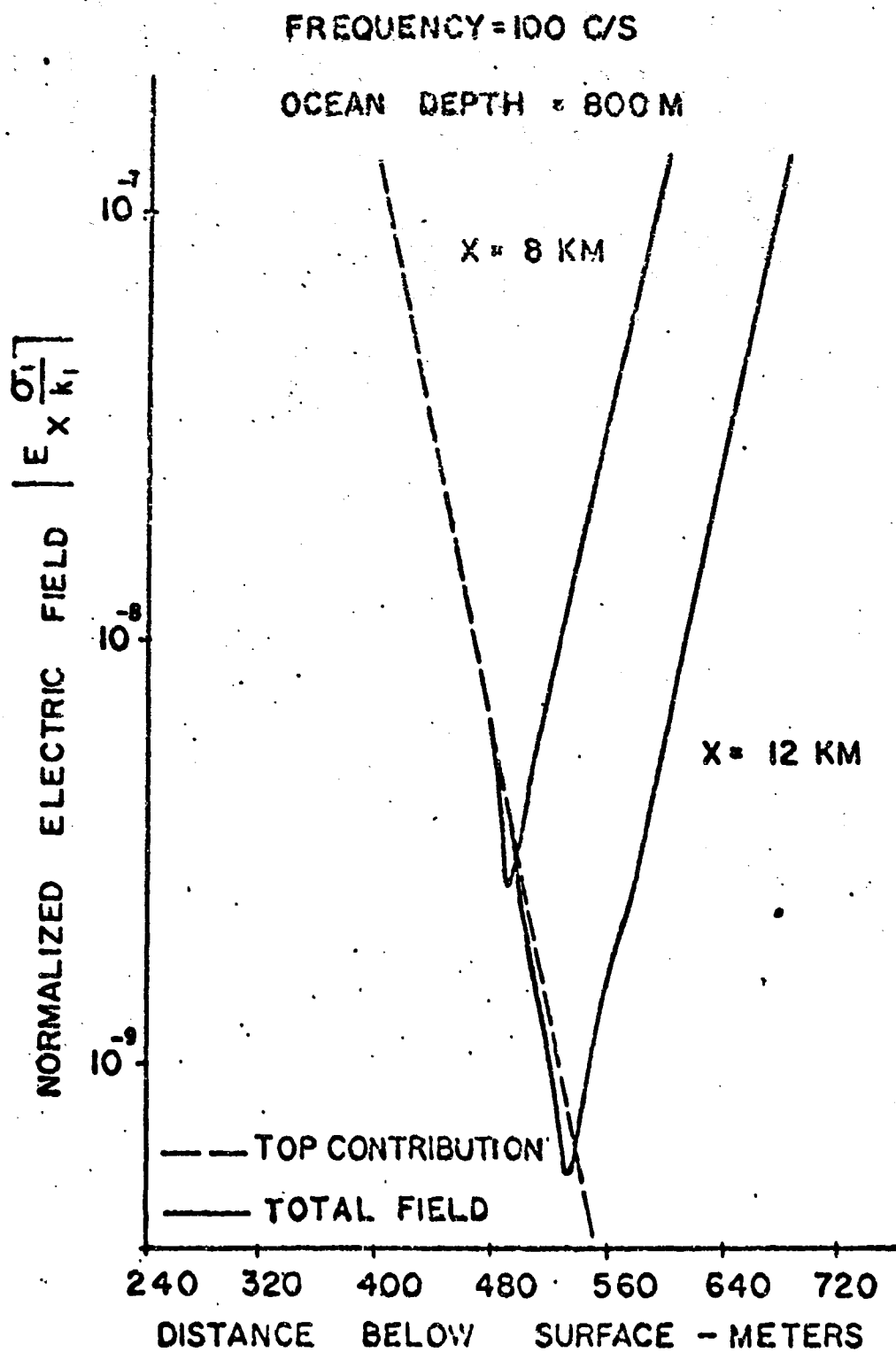


FIGURE - 16



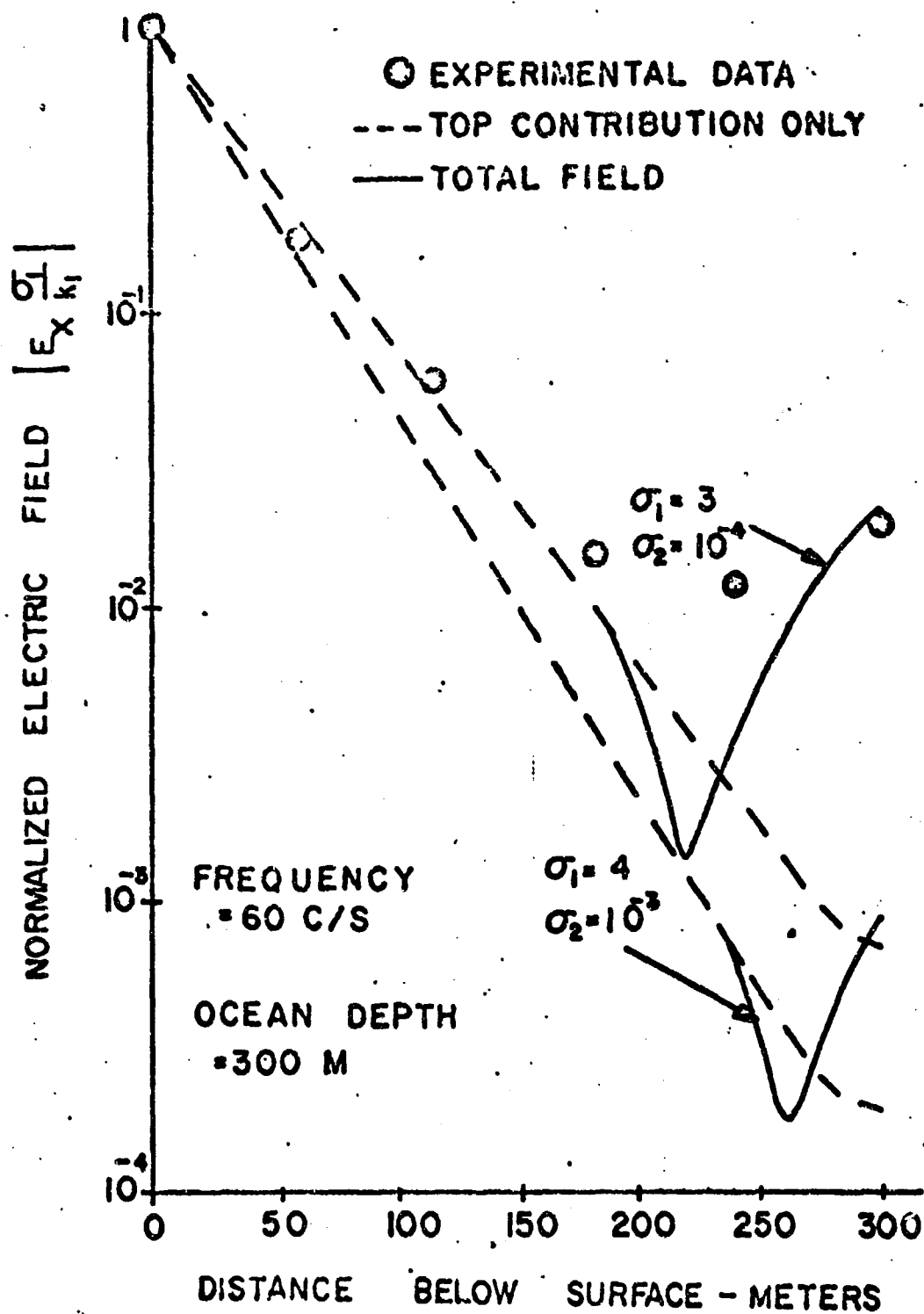


FIGURE - 17

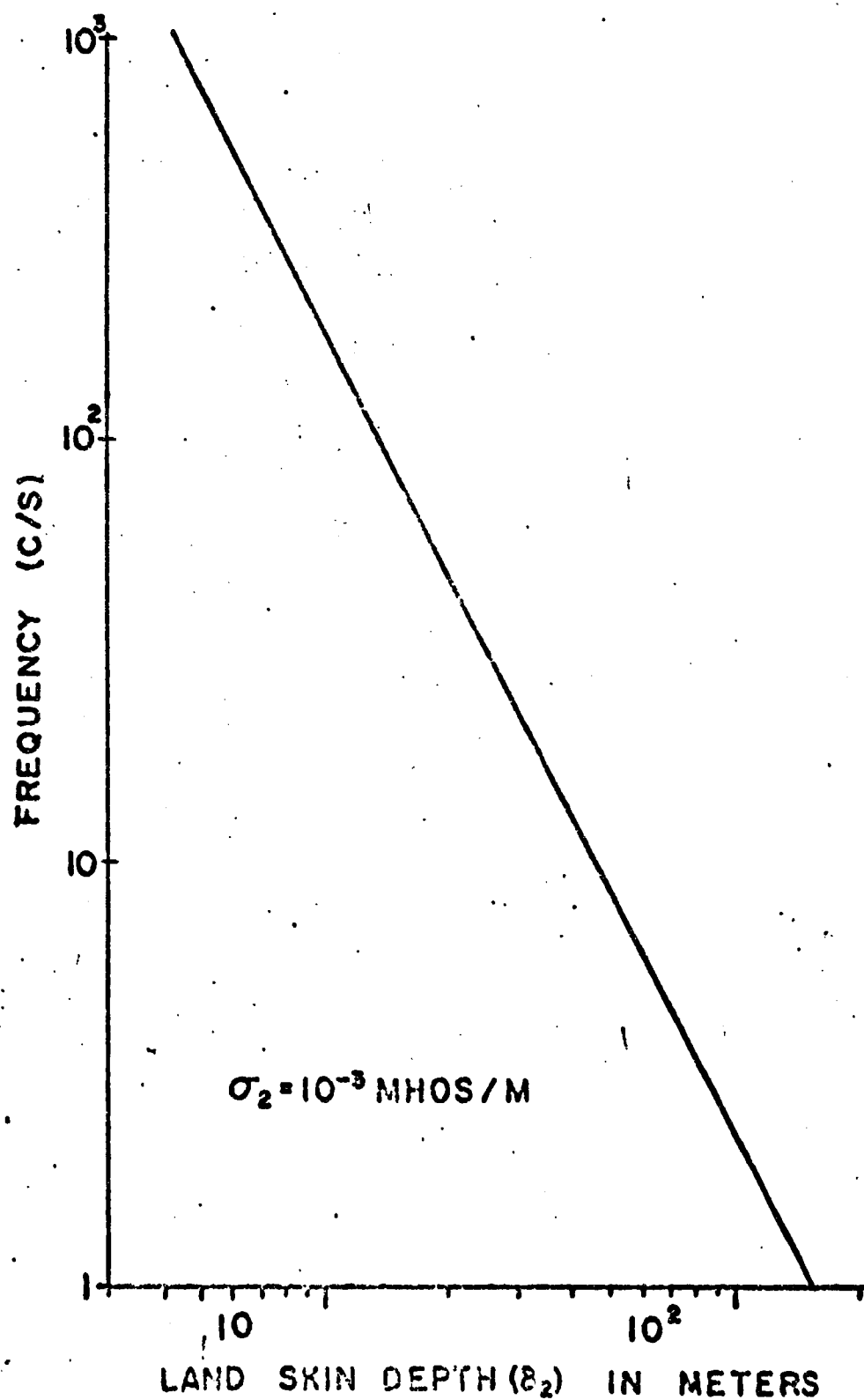
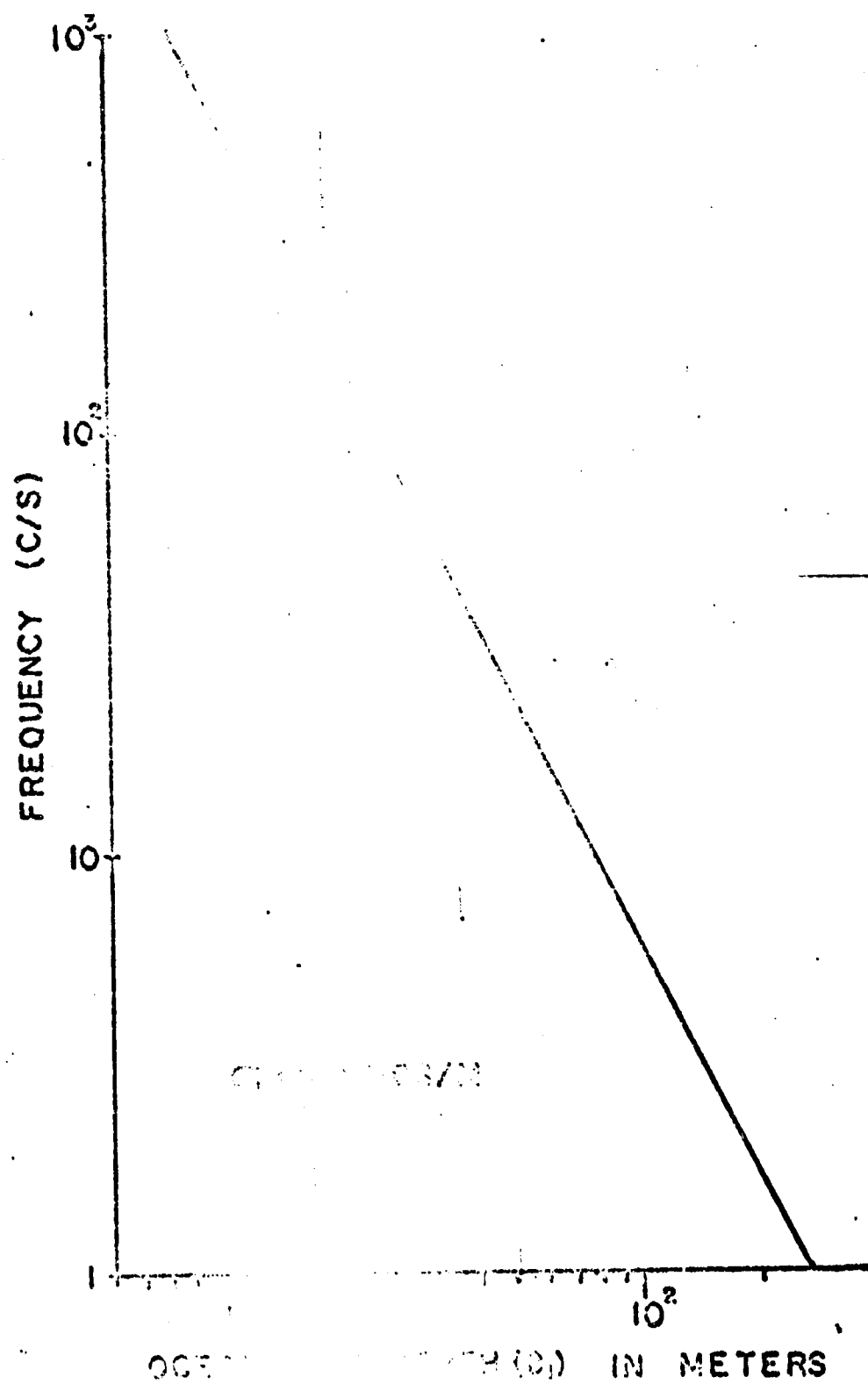


FIGURE--18

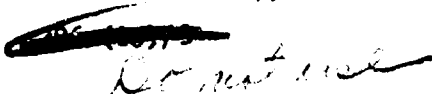


C/S = 1000/10

FIGURE--10

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<p>This report represents an extension of the work described in an earlier report [Electromagnetic Fields in the Ocean Near a Shoreline, J.E. Spence, E. Sullivan and J. Beville - Contract Nonr 396(10)/2 - 15 November 1965]. It is an analytical study of the electromagnetic field in the ocean resulting from excitation over land by natural electromagnetic noise such as micropulsations or ELF atmospherics. The main purpose of the study is to determine the relative importance of electromagnetic energy entering the ocean via the air-water interface versus energy entering via the soil and ocean bottom. Emphasis in this report is placed on a comparison between the vertical and horizontal components of the electric field. Vertical profiles of the horizontal and vertical electric field vectors are plotted for several values of frequency, ocean depth, and the distance from the shoreline. Finally, a comparison is made with some experimentally determined values of the horizontal electric field.</p>		

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	ROLE	WT	ROLE	WT	ROLE	WT
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